Estimation of Arrival Time of a Signal

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Abstract

This paper presents our investigation into the typical problem of estimating a signal immersed in noise as well as its unknown arrival time. The vector space theory has been effectively applied to the problem. The applied algorithm estimates the signal and its arrival time when only the orthonormal basis of signal subspace is known. The problem has been investigated for two situations. First, when any of the infinite signals from the signal subspace can be transmitted. Second, when a signal from particular set of known signals can be transmitted, as is the case with different Communication systems. The simulation results show that the standard deviation of our estimation is inversely related to the SNR of the received signal, whereas it is independent of amount of delay i.e the arrival time of the signal. The results are based on Monte Carlo simulations. The noise has been taken as AWGN throughout.

1 Introduction and Motivation

The problem of Time Delay Estimation has always been of great importance to researchers involved in communications and Signal Processing. This estimation of Arrival Time of the signal which may be a pulse or a sequence of pulses is of critical importance to many applications, specially where synchronization is required. Examples include radars, sonars and range finding equipment etc.

The problem has previously been investigated in depth because of the numerous applications that require a good estimate of the arrival time of the desired signal. Kay [1] has summarized some of the techniques employed for the said purpose, almost all of them require manipulation of joint and conditional Probability Density Functions. As shown in [1] many of the times the solution becomes intractable specially when the dimensions of the observation space increase and the noise does not remain white Gaussian.

Another approach has been pursued by Proakis [2] who has used the notion of Minimum Distance Decoding by reducing the MLE and MAP estimators to the distance between the observation and original signal. The problem of signal estimation has been discussed in detail in [2] but the estimation of arrival time has generally been ignored.

This is one of the problems that always need a better solution than the existing one, this is why we feel encouraged to present our solution.

In this report we have presented an algorithm based on the vector space theory and have used it to detect and estimate a signal and its arrival time, where the signal is from a given subspace immersed in noise from a larger space. The algorithm uses the concept of Projections and depending upon the quality of our signal estimation, gives us the estimate of delay of the signal. The delay has been calculated with respect to the time of reception of first sample. It is assumed that the orthonormal basis of the subspace of interest which were used to construct the signal are known to the receiver.

There are many factors that govern the quality of estimation of the signal and performance of the algorithm, namely dimension of observation space, dimension of signal subspace, length of the signal, the amount of delay and most importantly the SNR. In this paper we present our results on the effect of SNR and the delay on the performance of our estimation algorithm.

The second case involves the transmission of a finite number of known signals. Most of the algorithm is same as the first case, and a little modification in the final stage accomplishes the required task. This case is a logical modification of the first case. In this paper we have developed the theory and algorithm for second case but do not provide separate results for performance evaluation of the algorithm.

The outline of this paper is as follows.
In Section 2, we give the mathematical derivation of our framework. Section 3 presents the simulation results, and conclusions are summarized in Section 4.

2 Mathematical Framework

A subspace of dimension DimS has been generated which lies in a larger space of dimension DimV. The orthonormal basis D of length lenS that span our signal subspace are known to the receiver. An infinite number of signals lie in this subspace. A random set of coefficients α has been chosen to generate a signal S from the subspace. $S \in Span(D)$

This signal S is then subjected to a delay of n_o and is then immersed in Additive White Gaussian Noise N of dimension DimV. Resulting signal is the observation Y which is received at the receiver.

$$S = \sum_{i=1}^{dimS} \alpha_i D_i = \alpha^H D =< D, \alpha > \qquad (1)$$

$$Y = S[n - n_o] + N \qquad (2)$$

$$n_o \in [0, Td]$$

At the receiver we get an observation which has dimensions DimV and length DimV. This observation carries a signal of length lenS somewhere in-between. The possible values of n_o lie in the interval [0,Td] where $n_o = 0$ denotes zero delay and

$$Td = DimV - lenS - 1$$

To estimate the signal that was transmitted we take the projection of the observation on our signal subspace by taking the inner product of Y with orthonormal basis for each sample in the interval [0,Td]. This gives us an estimate of the coefficients for corresponding delay. These coefficients are a sum of original coefficients corrupted with noise $\bar{N}$ that lies in the same signal subspace.

$$\hat{\alpha}_i =< Y_i, D >= \alpha_i + \bar{N} \tag{3}$$

Where Y_i denotes the samples of Y in the interval [i,i+lenS-1] and $\hat{\alpha}_i$ denote the estimate of corresponding vector of coefficients. These are then used to calculate the projection of the observation on the subspace. Thus an estimated signal is constructed.

$$\hat{S}_i =< D, \hat{\alpha}_i >= \hat{\alpha}_i^H D \tag{4}$$

$$\hat{S}_i =< Y_i, D > D$$

$$\forall i \in [0, Td]$$

As a result we get an estimate of signal for every possible delay i.e we get a total of Td+1 projected signals. We then find the correlation of each projection with its corresponding position in the observation. The correlation can be found by simply taking the inner product of both. We find the point at which the correlation is maximum. This point gives us the estimate of delay $\hat{n}_o$ in the arrival of the signal and the corresponding signal is the estimate of the original signal.

$$\hat{S} = argmax_{\hat{S}_i}(< Y_i, \hat{S}_i >) \tag{5}$$

$$\forall i \in [0, Td]$$

The $\hat{S}$ is the estimate of the signal and corresponding i is the estimate of the delay n_o i.e the arrival time of the signal.

In the above case we estimated the signal and delay for any of the infinite signals that lie in the signal subspace. The second case is when we have only a finite number of known signals K_i $i \in [1, M]$ that can be transmitted over the channel (as in most of the communication systems). Here M is the total number of signals. After finding the delay $\hat{n}_o$ and projection $\hat{S}$ of Y on our signal space we find its correlation with each of the possible signals K_i. As a result the signal that gives the maximum correlation with $\hat{S}$ is chosen as the received signal.

$$\hat{K} = argmax_{K_i}(< K_i, \hat{S} >) \tag{6}$$

$$\forall i \in [0, M]$$

In doing so we implicitly divide our signal space into smaller subspaces, one for each signal, and the correlation tells us the subspace in which the signal lies. Thus the corresponding signal is decided.

3 Simulation results

For the analysis presented the simulation parameters taken are DimV=100, DimS=3, lenS=20. Different values of AWGN variances and Arrival Times n_o have been taken to study their effect on the performance of our algorithm. The algorithm has been tested using Monte Carlo Simulations with 1000 simulations per SNR per Delay. A random signal subspace has been generated and its orthonormal basis have been found using Gram Schmidt orthonormalization procedure. A random vector α has been generated which then generates the signal. The signal is then given an arbitrary delay n_o and noise is added to it. At the receiving side the algorithm is applied and the Delay $\hat{n}_o$ and Signal $\hat{S}$ are estimated.

Figure 1 shows the histogram of estimated delay $\hat{n}_o$ of the signal when original delay is set to 39 samples and variance of noise is 0.4. We notice that the histogram follows a Gaussian function. The center of the function is the actual delay n_o, and the spread denotes the erroneous

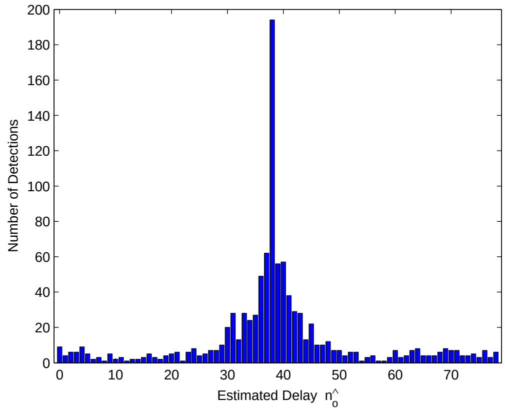

Figure 1: Histogram of estimated delay $\hat{n}_o$ of the signal; original delay=39; variance=0.4

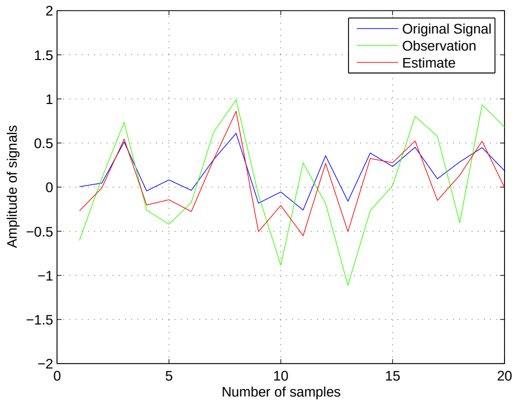

Figure 2: The original signal, the observation and the projection.

decisions. As the SNR increases the spread decreases.

Figure 2 shows the original signal, the corresponding interval of the observation and the resulting projection of the observation on the signal subspace. This figure is taken from a realization when the signal was detected correctly. Some correlation can be seen between the signal and projection.

Figure 3 shows the correlation between all the

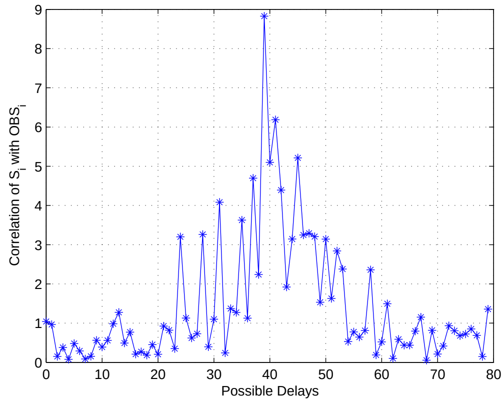

Figure 3: The correlation between observation and the set of projections; n_o=39. maximum=39.

projections and corresponding samples of observation which give a maximum at the original value of n_o. Thus n_o is estimated correctly. However if the maximum is not at n_o, we get a false estimate.

Figure 4 shows the change in standard deviation of the estimated delay $\hat{n}_o$ as dependent upon the variance of the AWG noise. As the noise variance increases the standard deviation increases and then gets virtually constant.

Figure 5 shows the standard deviation of the estimated delay $\hat{n}_o$ for different values of delays n_o and different noise variances. We can see that for a fixed noise variance, the delay in the signal arrival time n_o does not effect the performance of the algorithm. As the number of simulations grow large, the constant variance lines gets more and more straightened.

4 Conclusions and Future Work

We have presented a simple idea for signal estimation and finding its arrival time using the

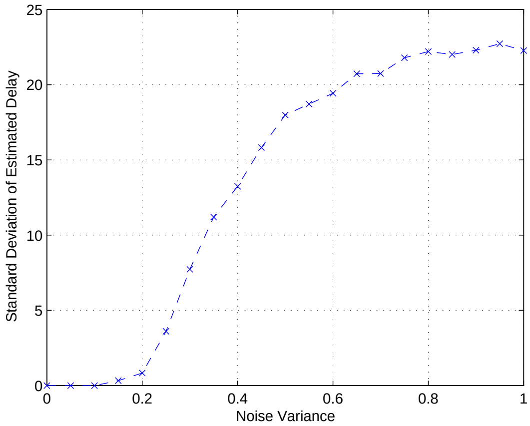

Figure 4: Dependence of standard deviation of $\hat{n}_o$ on noise variance.

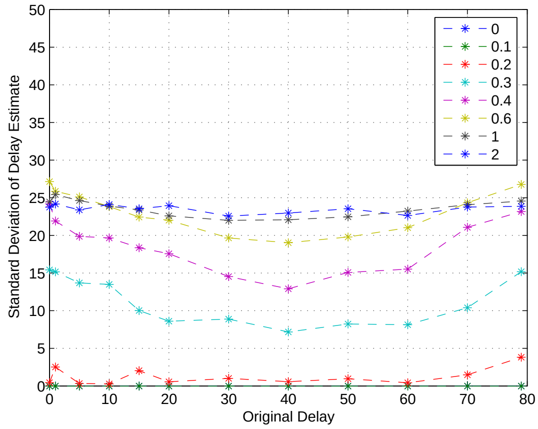

Figure 5: Standard deviation of estimate $\hat{n}_o$ for different delays n_o and noise variances; Simulations=1000 per delay per noise variance.

powerful tools provided by vector space theory. We have discussed two cases of estimation. First case deals with the estimation of any signal arriving from a specific subspace, holding infinite number of signals. In second case we estimate the signal and delay when only known signals can be transmitted so we have to decide which of the signals was transmitted. We assume prior knowledge of orthonormal basis used to generate the signals.

It has been shown that the performance of our algorithm is independent of amount of delay and improves as the SNR of observation improves.

The algorithm is computationally very intense, in our future work we will try to reduce the number of computations by identifying any redundant computations that waste precious resources.

The algorithm can be tailored to better suit any modifications and requirements at the transmitter or receiver side. It can also be modified to detect the presence or absence of a signal as well, by employing thresholding in the final stage i.e after calculating the correlation. However the design of optimal threshold needs further study.

5 References

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